Graph the curve

$$r = 1 + \cos\frac{\theta}{2}, \quad 0 \le \theta \le 4\pi,$$

in the x-y plane.

Compute the area bounded by the curve.

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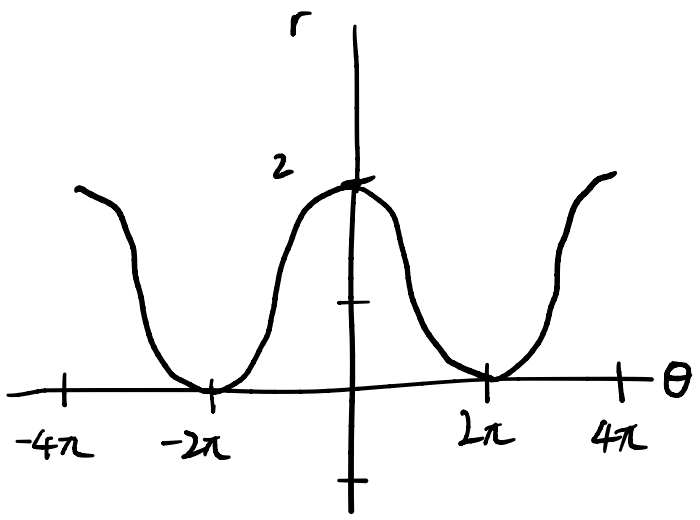

θ	r	x	y
0	2	2	0
π	1	-1	0
2π	0	0	0
3π	1	-1	0
4π	2	2	0
$\frac{\pi}{2}$	$1+\frac{\sqrt{2}}{2}$	0	$1+\frac{\sqrt{2}}{2}$
$\frac{3\pi}{2}$	$1-\frac{\sqrt{2}}{2}$	0	$-\left(1-\frac{\sqrt{2}}{2}\right)$
$\frac{5\pi}{2}$	$1-\frac{\sqrt{2}}{2}$	0	$1-\frac{\sqrt{2}}{2}$

$\therefore$ Total Area

$= 2\left(\frac{3}{4}\pi + 2 - \left(\frac{\pi}{4} - 2\right)\right) + 2\left(\frac{3\pi}{4} - 2\right) = \frac{3\pi}{2} + 4$

Compute the area bounded by the curve.

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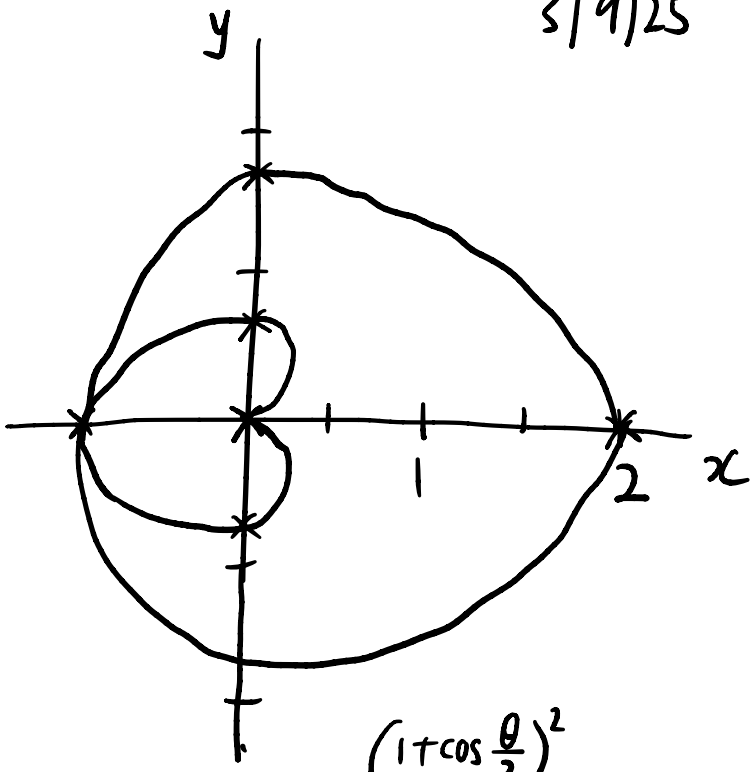

$\frac{dA}{\pi r^2} = \frac{r\,d\theta}{2\pi r}$

$dA = \frac{1}{2} r^2 \, d\theta$

$\left(1+\cos\frac{\theta}{2}\right)^2$

$= 1 + 2\cos\frac{\theta}{2} + \cos^2\frac{\theta}{2}$

$= 1 + 2\cos\frac{\theta}{2} + \frac{1+\cos\theta}{2}$

$= \frac{3}{2} + 2\cos\frac{\theta}{2} + \frac{\cos\theta}{2}$

$$\text{Area}_1 = \int_0^{\pi} \frac{1}{2}\left(1+\cos\frac{\theta}{2}\right)^2 d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \frac{3}{2} + 2\cos\frac{\theta}{2} + \frac{\cos\theta}{2}\, d\theta$$

$$= \frac{1}{2}\left(\frac{3}{2}\theta + \underbrace{2\sin\frac{\theta}{2}}_{1/2} + \frac{\sin\theta}{2}\right)\Bigg|_0^{\pi}$$

$$= \frac{1}{2}\left(\frac{3}{2}\pi + 4 + 0\right)$$

$$= \frac{3}{4}\pi + 2$$

$$\text{Area}_2 = \frac{1}{2}\left(\frac{3}{2}\theta + 4\sin\frac{\theta}{2} + \frac{\sin\theta}{2}\right)\Bigg|_{\pi}^{2\pi}$$

$$= \frac{1}{2}\left(3\pi - \left(\frac{3}{2}\pi + 4(1)\right)\right)$$

$$= \frac{3\pi}{4} - 2$$